JOURNAL OF MATERIALS SCIENCE 22 (1987) 1390–1396

Theoretical calculation of creep and relaxation of polycrystals, and stress redistribution among constituent grains

Z. G. ZHU, G. J. WENG
Department of Mechanics and Materials Science, Rutgers University, New Brunswick, New Jersey 08903, USA

Taking into account both transient and steady creep of slip systems in the grain, a theoretical method is developed to determine the overall creep and relaxation behaviour of polycrystals and, by which, the accompanying stress and strain distribution among the constituent grains can also be evaluated. This method extends the incremental self-consistent relation for grain interactions to the total form, and is further complemented with an iterative computational process. It is primarily intended for the calculation of creep under a constant stress, relaxation under a constant strain, and a combination of both. While maintaining almost the same degree of accuracy, this new method, as compared to the incremental one, is far more effective. Its theoretical predictions on the creep and relaxation of a 2618-T6I aluminium are shown to be in good accord with experiments. The heterogeneous nature of creep deformation and stress distribution among the constituent grains are also displayed for several selected grain orientations. Finally some implications and limitations of the model are assessed.

1. Introduction

When a polycrystalline metal is subjected to a constant stress at elevated temperature, creep deformation takes places. While ostensibly deforming at a constant stress, stress distribution inside the polycrystal, primarily due to the variation of grain orientations, is highly heterogeneous. The truth is that the more favourably oriented grains will deform more extensively than the less favourably oriented ones, and the stress relieved by the former group will have to be carried over by the latter. The process of stress redistribution is time-dependent. As the creep activity of a constituent grain depends directly on its local stress, it is essential that such a distribution be characterized. Then the creep behaviour of the constituent grains and of the aggregate can be determined accordingly.

Instead of being submitted to a constant stress, the polycrystal may be subjected to a fixed total strain, resulting in a state of stress relaxation. The latter apparently occurs as a consequence of creep deformation. As creep caused by the initial stress in the constituent grains will lead to an increase in overall strain, the external stress has to be reduced continuously to compensate for such an increase. Both creep deformation and stress relaxation are not uniform within the aggregate, and we again have an inhomogeneous state of stress and strain among the constituent grains during the relaxation process.

Directed towards the solution of these problems, we present a simple method to evaluate the corresponding stress redistribution among the constituent grains, and from these to calculate the history of creep deformation and stress relaxation for both the grains and the aggregate. Since a material element in service may encounter a simultaneous creep and relaxation, the consideration will also be extended to such a simultaneous process, say combined creep in torsion and relaxation in tension. The proposed method stems from a general "incremental" method suggested by Weng [1], and is primarily intended for the cases of a polycrystal under the influence of *constant* stress and/or total strain as stated above. The rationale for this development is two-fold. First, the time increment adopted in the incremental method, to ensure numerical stability, has to be sufficiently small and this usually results in long computations. For instance in a recent study [2] the second writer had to calculate 3% of creep strain incrementally for aluminium, and it took more than 2000 steps. Keeping in mind that this was done for a polycrystal which consists of a large number of constituent grains, and each grain further possesses twelve slip systems, the computation was quite formidable indeed. Secondly, as creep and relaxation under constant stress or strain are frequently encountered in fundamental theory and experiment, there is an urgent need to develop a more efficient method. In contrast to the incremental method, the new one is developed in a "total" form at a given time. This idea grew out of the observations that, within the range of transient and steady creep, both creep and relaxation curves are monotonic, and may be well represented by three to five points. The new method thus aims at calculating the total creep strain and/or

 0022–2461/87 $03.00 + .12

relaxed stress at time t, without going through the small incremental steps.

2. Modification of the incremental self-consistent relation to the total form

Our first task is to establish the required principle of stress redistribution among the constituent grains. With the aid of Eshelby's [3] solution for a spherical inclusion, this can be conveniently accomplished by the self-consistent relation. Let the stress and creep strain of a constituent grain (spherical inclusion) be denoted by $\sigma_{ij}(t)$ and $\varepsilon^c_{ij}(t)$, and those of the polycrystalline aggregate (surrounding matrix) by the corresponding barred (averaged) quantities $\bar{\sigma}_{ij}(t)$ and $\bar{\varepsilon}^c_{ij}(t)$, respectively. We now modify the incremental self-consistent relation to the total form, and, for clarity, this will be carried out for creep under a constant stress, relaxation under a constant total strain, and simultaneous creep in torsion and relaxation in tension, in turn.

2.1. Creep under a constant stress

During an increment of creep, creep strains $d\varepsilon^c_{ij}$ and $d\bar{\varepsilon}^c_{ij}$ are developed in both the constituent grain and the aggregate, and this results in a stress redistribution. By identifying such a deformation as a truely "stress-free" process in the sense of Eshelby [3], the stress variation in the inclusion was shown to be [1]

$$d\sigma_{ij} = -2\mu(1-\beta)(d\varepsilon^c_{ij} - d\bar{\varepsilon}^c_{ij}), \qquad (1)$$

where μ is the elastic modulus and, in terms of Poisson's ratio ν, $\beta = 2(4-5\nu)/15(1-\nu)$. For simplicity the elastic property of the constituent grain is taken to be isotropic, and within the low and intermediate temperature range, creep deformation involves no significant volume change.

For a randomly oriented polycrystal, the creep strain is simply the average taken over all, say N, grain orientations; one has

$$d\bar{\varepsilon}^c_{ij} = \frac{1}{N}\sum_{p=1}^{N} d\overset{(p)}{\varepsilon}{}^c_{ij}. \qquad (2)$$

Obviously when Equation 1 is considered for all grains their sum will vanish, thereby satisfying the requirement of self-consistency.

Recognizing that the total strain consists of elastic and creep strains and that the perturbed elastic strain associated with such a process is $\beta(d\varepsilon^c_{ij} - d\bar{\varepsilon}^c_{ij})$, the increase of total strain is given by

$$d\varepsilon_{ij} = d\bar{\varepsilon}^c_{ij} + \beta(d\varepsilon^c_{ij} - d\bar{\varepsilon}^c_{ij}), \qquad (3)$$

for a constituent grain, and, for the aggregate, it is simply

$$d\bar{\varepsilon}_{ij} = d\bar{\varepsilon}^c_{ij}. \qquad (4)$$

Since the externally applied stress $\bar{\sigma}_{ij}$ is held constant, upon integration one has

$$\sigma_{ij}(t) = \bar{\sigma}_{ij} - 2\mu(1-\beta)[\varepsilon^c_{ij}(t) - \bar{\varepsilon}^c_{ij}(t)] \qquad (5)$$

$$\varepsilon_{ij}(t) = \bar{\varepsilon}_{ij}(0) + \beta\varepsilon^c_{ij}(t) + (1-\beta)\bar{\varepsilon}^c_{ij}(t) \qquad (6)$$

$$\bar{\varepsilon}^c_{ij}(t) = \frac{1}{N}\sum_{p=1}^{N} \overset{(p)}{\varepsilon}{}^c_{ij}(t) \qquad (7)$$

where $\bar{\varepsilon}_{ij}(0)$ is the initial strain corresponding to $\bar{\sigma}_{ij}$. It is evident from Equation 5 that the stress of a more favourably oriented grain ($\varepsilon^c_{ij} > \bar{\varepsilon}^c_{ij}$) will continue to decrease whereas that of a lesser one will continue to increase in the course of creep deformation. This self-consistent relation, with creep strains replaced by the corresponding plastic strains, was originally derived by Kröner [4] and Budiansky and Wu [5] for time-independent plasticity and its incremental form was first applied by Brown [6] to creep. While plastic strain is directly linked to stress in plasticity, the constitutive equation in creep also involves the creep rate, in addition to creep strain and stress. This additional dependence makes the application of Equation 5 more involved.

2.2. Relaxation under a constant total strain

Contrary to creep, relaxation is a "strain-free" stress process and its incremental self-consistent relations have been established by Weng [7]. It was shown that, during a time interval dt, the stress and strain redistribution among the constituent grains are characterized by

$$d\sigma_{ij} = -2\mu(1-\beta)(d\varepsilon^c_{ij} - d\bar{\varepsilon}^c_{ij}) - 2\mu d\bar{\varepsilon}^c_{ij} \qquad (8)$$

$$d\varepsilon_{ij} = \beta(d\varepsilon^c_{ij} - d\bar{\varepsilon}^c_{ij}) \qquad (9)$$

where the incremental creep strains were generated during dt. To maintain a fixed total strain, the relaxed stress of the polycrystal is

$$d\bar{\sigma}_{ij} = -2\mu d\bar{\varepsilon}^c_{ij} \qquad (10)$$

Keeping the total strain $\bar{\varepsilon}_{ij}$ constant, integrations of these equations lead to

$$\sigma_{ij}(t) = \bar{\sigma}_{ij}(0) - 2\mu(1-\beta)[\varepsilon^c_{ij}(t) - \bar{\varepsilon}^c_{ij}(t)] - 2\mu\bar{\varepsilon}^c_{ij}(t) \qquad (11)$$

$$\varepsilon_{ij}(t) = \bar{\varepsilon}_{ij} + \beta[\varepsilon^c_{ij}(t) - \bar{\varepsilon}^c_{ij}(t)] \qquad (12)$$

$$\bar{\sigma}_{ij}(t) = \bar{\sigma}_{ij}(0) - 2\mu\bar{\varepsilon}^c_{ij}(t) \qquad (13)$$

where $\bar{\sigma}_{ij}(0)$ is the initial stress, related to $\bar{\varepsilon}_{ij}$ through the isotropic elastic relation. Equation 12 indicates that, although the total strain of the specimen is fixed, those of its constituent grains, depending on their orientations, will continue to increase or decrease. Comparison between Equations 11 and 13 show that for favourably oriented grains the stress relaxes more rapidly than the overall specimen, whereas stress relaxation is slower in the less favourably oriented grains.

2.3. Simultaneous creep in torsion and relaxation in tension

When the material is subjected to a constant stress in one direction and constant strain in the other, simultaneous creep and relaxation take place. Such a behaviour is interactive. For instance many experiments [8, 9] have indicated that when a cyclic torsional strain is superimposed upon a constant tensile stress, it may lead to a tensile creep acceleration. Due to the availability of experimental data we consider here the specific case of simultaneous creep in torsion and relaxation in tension; other types of mixed loading may be analysed in a similar fashion.

Now that $\bar{\sigma}_{12}$ and $\bar{\varepsilon}_{11}$ are kept constant, with other $\bar{\sigma}_{ij} = 0$, the stress and strain redistribution of the constituent grains may be obtained by the combination of the foregoing considerations. Thus

$$\sigma_{11}(t) = \bar{\sigma}_{11}(0) - 2\mu(1-\beta)[\varepsilon_{11}^{c}(t) - \bar{\varepsilon}_{11}^{c}(t)] - E\bar{\varepsilon}_{11}^{c}(t) \quad (14)$$

$$\sigma_{ij}(t) = \bar{\sigma}_{ij} - 2\mu(1-\beta)[\varepsilon_{ij}^{c}(t) - \bar{\varepsilon}_{ij}^{c}(t)] \quad (i,j \neq 1) \quad (15)$$

$$\varepsilon_{11}(t) = \bar{\varepsilon}_{11} + \beta[\varepsilon_{11}^{c}(t) - \bar{\varepsilon}_{11}^{c}(t)] \quad (16)$$

$$\varepsilon_{ij}(t) = \bar{\varepsilon}_{ij}(0) + \beta\varepsilon_{ij}^{c}(t) + (1-\beta)\bar{\varepsilon}_{ij}^{c}(t) + \nu\delta_{ij}\bar{\varepsilon}_{11}^{c}(t) \quad (i,j \neq 1) \quad (17)$$

$$\bar{\sigma}_{11}(t) = E[\bar{\varepsilon}_{11} - \bar{\varepsilon}_{11}^{c}(t)] \quad (18)$$

where E is the Young's modulus and δ_{ij} the Kronecker delta (1 when $i = j$, and 0 when $i \neq j$). The change of 2μ in Equation 11 to E in Equations 14 and 18, and the presence of the last term in Equation 17, are attributed to the fact that $\bar{\sigma}_{22} = \bar{\sigma}_{33} = 0$.

3. Creep deformation of constituent grains and slip systems

We shall restrict our consideration to the high stress, low to intermediate temperature range where dislocation glide, or simply crystallographic slip, may qualify as the principal source of creep deformation. The creep strain of a constituent grain is now contributed by the time-dependent slip strains of its slip systems. The slip activity of a slip system initially depends on its resolved shear stress τ, and, due to both active and latent hardening, its creep rate gradually decreases to a steady state. Using the simple power law for its stress-dependence the steady and transient creep rates of, say the kth slip system, may be respectively written as [1]

$$\overset{(k)}{\dot{\gamma}}{}_{s}^{c} = \kappa \overset{(k)}{\tau}{}^{\lambda} \quad (19)$$

$$\overset{(k)}{\dot{\gamma}}{}_{t}^{c} = \eta\left\{\zeta\overset{(k)}{\tau}{}^{\lambda} - \sum_{l=1}^{n}\left[\alpha + (1-\alpha)\cos\overset{(k,l)}{\theta}\cos\overset{(k,l)}{\phi}\right]\overset{(l)}{\gamma}{}^{c}\right\} \quad (20)$$

where κ, λ, η, ζ and α are material constants, $\overset{(k,l)}{\theta}$ and $\overset{(k,l)}{\phi}$, respectively, the angles between the slip directions and slip-plane normals of the kth and lth systems. These material constants, as will be shown later, may be determined from two tensile creep tests of the polycrystal, one followed by strain recovery. Among these, λ characterizes the stress-dependence, or the separation of two creep curves under two different stresses, κ controls the steady creep rate, ζ gives the magnitude of transient creep, η controls the decreasing rate of creep rate, and α represents the internal back-stress. When $\alpha = 1$, the latent hardening reduces to Taylor's [10] isotropic hardening and when $\alpha = 0$, it corresponds to Prager's kinematic hardening [11]. In general, its value is less than 1, giving rise to strain recovery upon unloading. As the resolved shear stress τ of a slip system directly depends on the local stress $\sigma_{ij}(t)$ of the constituent grain to whch it belongs, its value also continues to change during a creep or relaxation process.

The total creep rate of a slip system is the sum of the steady and transient rates; thus

$$d\gamma^{c} = (\dot{\gamma}_{s}^{c} + \dot{\gamma}_{t}^{c})\,dt \quad (21)$$

Since creep deformation may take place simultaneously in all slip systems, the creep strain of a constituent grain is given by

$$d\varepsilon_{ij}^{c} = \sum_{k=1}^{n}\overset{(k)}{\nu}_{ij}\,d\overset{(k)}{\gamma}{}^{c} \quad (22)$$

where the summation extends to all n systems, and $\overset{(k)}{\nu}_{ij}$ is the Schmid-factor, or orientation-factor tensor of the kth system, given by its unit slip direction b_i and slip-plane normal n_i by

$$\overset{(k)}{\nu}_{ij} = \tfrac{1}{2}(\overset{(k)}{b}_{i}\overset{(k)}{n}_{j} + \overset{(k)}{b}_{j}\overset{(k)}{n}_{i}) \quad (23)$$

Similarly at a given time t, the total creep strain of a constituent grain is given by

$$\varepsilon_{ij}^{c}(t) = \sum_{k=1}^{n}\overset{(k)}{\nu}_{ij}\overset{(k)}{\gamma}{}^{c}(t) \quad (24)$$

Looking back at the "total" self-consistent relations established in the preceding section, it is evident that the whole problem of creep and relaxation is now crucially tied to the determination of $\gamma^{c}(t)$ for all slip systems in all grains.

4. Determination of the creep strain $\gamma^{c}(t)$ of a slip system

Primarily due to the time-dependence of the resolved shear stress $\tau(t)$ and the coupling nature of creep strains in Equations 19 and 20, it is difficult to solve analytically for $\gamma^{c}(t)$ by direct integration. Our objective here is to propose a simple computational method for such a purpose.

The resolved shear stress of the kth slip system in the pth grain is in general given by

$$\overset{(p,k)}{\tau} = \overset{(p,k)}{\nu}_{ij}\overset{(p)}{\sigma}_{ij}, \quad (25)$$

where $\overset{(p)}{\sigma}_{ij}$ is the local stress of the pth grain at time t and, depending on the problem considered, is given by the appropriate self-consistent relation. The Einstein summation convention for a repeated index is implicitly adopted in this paper; thus the subscripts i and j are to be summed over 1 to 3 in Equation 25. It is now necessary to introduce the superscript p to designate a specific grain orientation.

Expressing the creep strain of the polycrystal by Equation 7 and further that of an individual grain by Equation 24, we can now use the various self-consistent relations derived earlier to write τ in terms of its initial value τ_0 and γ^{c}.

(i) Creep at constant $\bar{\sigma}_{ij}$: With $\overset{(p,k)}{\tau_0} = \overset{(p,k)}{\nu}_{ij}\bar{\sigma}_{ij}$ we have, from Equations 25 and 5,

$$\overset{(p,k)}{\tau}(t) = \overset{(p,k)}{\tau_0} - 2\mu(1-\beta)\overset{(p,k)}{\nu}_{ij}\left[\sum_{l=1}^{n}\overset{(p,l)}{\nu}_{ij}\overset{(p,l)}{\gamma}{}^{c}(t) - \frac{1}{N}\sum_{q=1}^{N}\sum_{l=1}^{n}\overset{(q,l)}{\nu}_{ij}\overset{(q,l)}{\gamma}{}^{c}(t)\right] \quad (26)$$

(ii) Relaxation at constant strain $\bar{\varepsilon}_{ij}$: With $\bar{\sigma}_{ij}(0) = 2\mu\bar{\varepsilon}_{ij} + \lambda\delta_{ij}\bar{\varepsilon}_{kk}$ (where μ and λ are the Lame' constants) and $\overset{(p,k)}{\tau_0} = \overset{(p,k)}{v_{ij}}\, \bar{\sigma}_{ij}(0)$, one finds from Equations 25 and 11

$$\overset{(p,k)}{\tau}(t) = \overset{(p,k)}{\tau_0} - 2\mu \overset{(p,k)}{v_{ij}} \left[(1-\beta) \sum_{l=1}^{n} \overset{(p,l)}{v_{ij}} \overset{(p,l)}{\gamma^c}(t) + \frac{\beta}{N} \sum_{q=1}^{N} \sum_{l=1}^{n} \overset{(q,l)}{v_{ij}} \overset{(q,l)}{\gamma^c}(t) \right] \quad (27)$$

(iii) Creep at constant $\bar{\sigma}_{12}$ and relaxation at constant $\bar{\varepsilon}_{11}$ (other $\bar{\sigma}_{ij} = 0$): With $\bar{\sigma}_{11}(0) = E\bar{\varepsilon}_{11}$, $\overset{(p,k)}{\tau_0} = \overset{(p,k)}{v_{11}}\, \bar{\sigma}_{11}(0) + \overset{(p,k)}{v_{12}}\, \bar{\sigma}_{12} + \overset{(p,k)}{v_{21}}\, \bar{\sigma}_{21}$, and Equation 14, 15 and 25, the resolved shear stress is given by

$$\overset{(p,k)}{\tau}(t) = \overset{(p,k)}{\tau_0} - 2\mu(1-\beta) \overset{(p,k)}{v_{ij}} \times \left[\sum_{l=1}^{n} \overset{(p,l)}{v_{ij}} \overset{(p,l)}{\gamma^c}(t) - \frac{1}{N} \sum_{q=1}^{N} \sum_{l=1}^{n} \overset{(q,l)}{v_{ij}} \overset{(q,l)}{\gamma^c}(t) \right] - \frac{E}{N} \overset{(p,k)}{v_{11}} \sum_{q=1}^{N} \sum_{l=1}^{n} \overset{(q,l)}{v_{11}} \overset{(q,l)}{\gamma^c}(t) \quad (28)$$

With the proper $\overset{(p,k)}{\tau}(t)$, the creep rate of the kth slip system in the pth grain depends only on its initial resolved shear stress, and the creep strains of all slip systems. We rewrite Equations 19 and 20 as

$$\overset{(p,k)}{\dot{\gamma}^c}(t) = (\kappa + \eta\zeta) \overset{(p,k)}{\tau^\lambda}(t) - \eta \sum_{l=1}^{n} [\alpha + (1-\alpha) \cos \overset{(k,l)}{\theta} \cos \overset{(k,l)}{\phi}] \overset{(p,l)}{\gamma}{}^c(t), \quad (29)$$

where the grain runs from $p = 1, 2 \ldots$, to N and in each grain the slip system goes from $k = 1, 2 \ldots$, to n. Equation 29 in effect represents a system of nN nonlinear, coupled differential equations in the form of

$$\overset{(p,k)}{\dot{\gamma}^c}(t) = F(\overset{(p,k)}{\tau_0}, \overset{(q,l)}{\gamma^c}(t); \quad q = 1, \ldots, N, \text{ and } l = 1, \ldots, n) \quad (30)$$

To illustrate the proposed method let us first consider the simple function with only one variable γ,

$$\dot{\gamma} = f(\gamma). \quad (31)$$

Referring to Fig. 1, suppose that γ_A at time t_1 is known and we wish to find γ_B at time t_2, such that the time difference $\Delta t = t_2 - t_1$ is not necessarily small. The strain γ_B can be obtained approximately by a simple iterative process. We first set $\gamma = \gamma_A$ in Equation 31 to compute the rate and find γ_1 as

$$\gamma_1 = \dot{\gamma}_A \Delta t + \gamma_A \quad (32)$$

This γ_1 is clearly greater than the true γ_B and, with which, a smaller creep rate $\dot{\gamma}_1$, may be computed from Equation 31 (corresponding to the rate at point 1′). We then find the strain from the second iteration as

$$\gamma_2 = \tfrac{1}{2}(\dot{\gamma}_A + \dot{\gamma}_1)\Delta t + \gamma_A \quad (33)$$

and this also gives us the creep rate $\dot{\gamma}_{2'} = f(\gamma_2)$. Repeating this iteration we have

$$\gamma_m = \tfrac{1}{2}(\dot{\gamma}_A + \dot{\gamma}_{(m-1)})\Delta t + \gamma_A \quad (34)$$

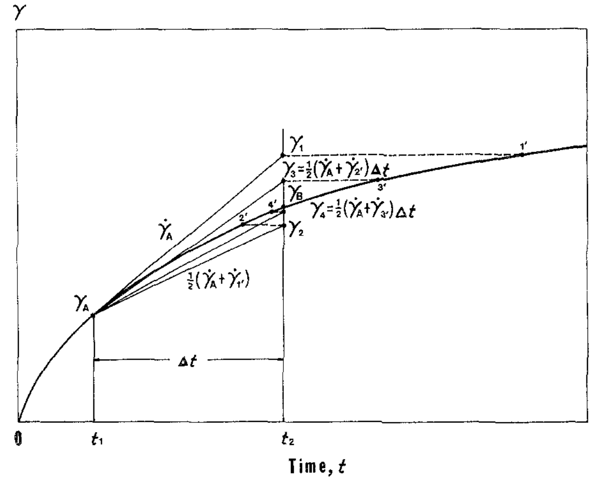

Figure 1 Schematic illustration of the iterative method.

after m iterations. Usually after five iterations the obtained γ would be sufficiently close to the γ_B calculated incrementally.

Now returning to our Equation 30, we note that $\gamma^c = 0$ initially for all slip systems and these serve as the initial points. Then the initial creep rates may be computed from Equation 30 or 29, for all systems in conjunction with the appropriate $\tau_0(t)$. Applying Equations 32 to 34 for each slip system, the value of $\overset{(p,k)}{\gamma^c}(t_1)$, at time t_1, may be obtained approximately after some iterations. We then use these $\overset{(p,k)}{\gamma^c}(t_1)$ values as the new starting points to repeat the same iterative process for $\overset{(p,k)}{\gamma^c}(t_2)$. Because creep rate decreases monotonically, an overestimate in $\gamma^c(t_1)$ would subsequently lead to an underestimate in the increment, $\gamma^c(t_2) - \gamma^c(t_1)$. Such a self-adjusting nature, as will become evident in our numerical results, further improves the accuracy of this method over the entire curve.

The proposed iterative scheme of course depends on a stable convergence. For the transient and steady creep, as the creep rate decreases monotonically such a convergence is always assured. During the steady state when the creep rate remains constant, this method is accurate without any iteration. However if the creep rate monotonically increases as in the tertiary creep, such a condition is assured only when $(\dot{\gamma}_A + \dot{\gamma}_B)/2 \leqslant (\gamma_B - \gamma_A)/\Delta t$.

5. Creep and relaxation of an aluminium alloy, and the accompanied stress redistribution

It is now of interest to apply this new method to predict the creep and relaxation behaviour of a practical system, and to compare the results with the original incremental calculations and experimental data. To this end we choose the 2618-T61 aluminium alloy, which has been systematically tested at 200° C by Ding and Findley [12] and Lai and Findley [13].

Aluminium has a face centred cubic crystal structure; it has four $\{1\,1\,1\}$ slip planes and three $\langle 1\,1\,0 \rangle$ slip directions on each plane, resulting in a total of twelve slip systems in each grain. The polycrystal model is chosen to consist of seventy-five different grain-orientations. This model, originally used in [1],

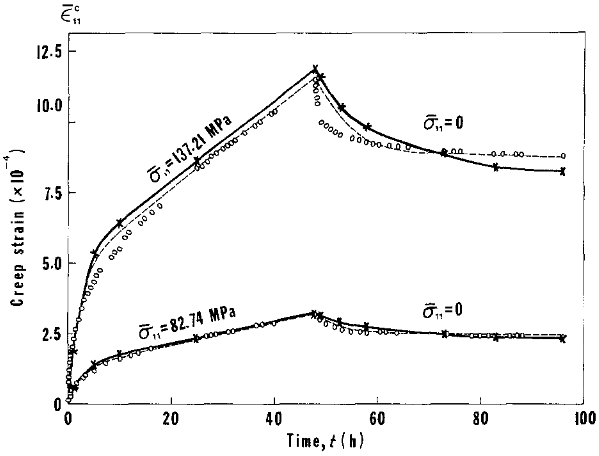

Figure 2 Derivation of micro parameters by the simulation of tensile creep and strain recovery. (x) Total method, (---) incremental method, (O) Ding and Findley [12].

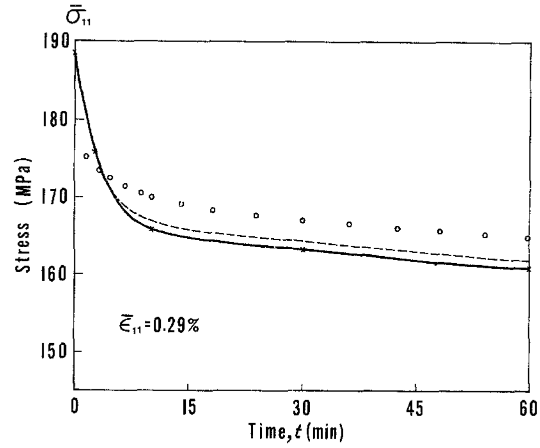

Figure 4 Theoretical predictions of tensile stress relaxation. (x) Total method, (---) incremental method, (O) Lai and Findley [13].

is reasonably isotropic; as judged from the generated creep strain components, the maximum deviation from isotropy is about 5%. Equation 30 involves therefore a total of 75 × 12 = 900 nonlinear, coupled differential equations. With these chosen orientations the six components of orientation-factor tensor v_{ij} for the slip systems, totalling 6 × 900 = 5400, may be stored in the computer for ready use.

Equation 29 involves five parameters; κ, η, ζ, λ and α, on the microscales, and these may be determined by an inverse simulation of two tensile creep and one strain-recovery curves. The two tensile creep curves, tested at $\bar{\sigma}_{11} = 82.74\,\text{MPa}$ (12×10^3 p.s.i) and 137.21 MPa (19.9×10^3 p.s.i.), and the corresponding recovery curves [12] are reproduced in Fig. 2. To be more accurate, we used the incremental method, which was proven to be reliable, to determine these micro properties. (As will become evident the values derived by the total method should not differ significantly.) With $E = 65\,\text{GPa}$ and $\mu = 23.8\,\text{GPa}$, resulting in $\nu = 0.366$, at the tested temperature [12], these simulations lead to $\kappa = 1.935 \times 10^{-10}$, $\lambda = 2.70$, $\eta = 0.20$, $\zeta = 1.168 \times 10^{-8}$, and $\alpha = 0.17$, where stress, strain and time are in the units of MPa, $\text{m}\,\text{m}^{-1}$ and h, respectively. These parameters were then used in the newly introduced total method to compute the corresponding creep and strain recovery. The last point in creep was taken as the first point in the recovery calculations, then setting $\bar{\sigma}_{ij} = 0$. With the cross sign x marking the selected points in this new method, the theoretical curves by both schemes are displayed in Fig. 2, along with the test data. While both methods clearly show the ability to simulate the observed behaviour, it is also evident that the results of this simple method follow closely those of the incremental one.

We further used these parameters to *predict* the creep and strain recovery under combined tension and shear, at $\bar{\sigma}_{11} = 82.74\,\text{MPa}$ (12×10^3 p.s.i.) and $\bar{\sigma}_{12} = 41.37\,\text{MPa}$ (6×10^3 p.s.i.). The predicted results by both methods and the experimental data are shown in Fig. 3. Though the shear component of the recovery strain appears to be overestimated, the theoretical results in the other cases appear to be in accord with experiments.

Ding and Findley [12] did not examine the relaxation behaviour in the same study, but such tests were carried out earlier by Lai and Findley [13]. As the chemical compositions of these two lots of aluminium alloy are slightly different, and the material used in [12] was received 14 years later, their creep behaviour also appear to be somewhat different. The material properties in [13] were derived previously as [1]: $\kappa = 2.52 \times 10^{-10}$, $\lambda = 4.12$, $\eta = 0.10$, $\zeta = 1.09 \times 10^{-7}$, and $\alpha = 0.28$, where stress, strain, and time are in the units of MPa, $10^{-4}\,\text{m}\,\text{m}^{-1}$, and min, respectively. Using these constants, the predicted stress relaxation under a constant tensile strain $\bar{\varepsilon}_{11} = 0.29\%$ (other $\bar{\sigma}_{ij} = 0$) are given in Fig. 4, along with

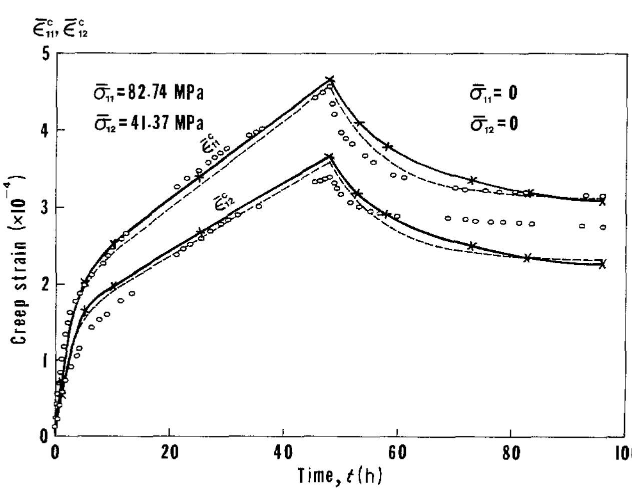

Figure 3 Theoretical predictions of creep strain and strain recovery under combined tension and torsion. (x) Total method, (---) incremental method, (O) Ding and Findley [12].

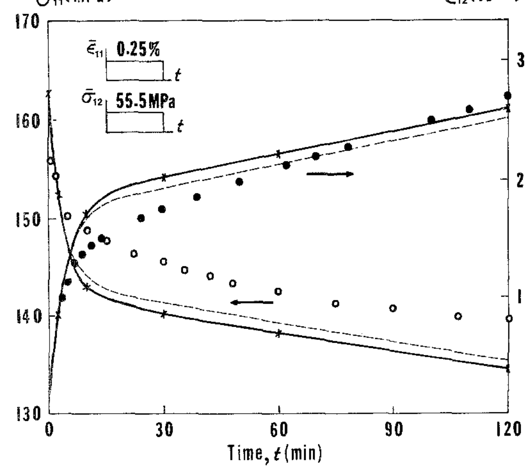

Figure 5 Theoretical predictions of simultaneous creep and relaxation. (x) Total method, (– – –) incremental method, (●, ○) Lai and Findley [13].

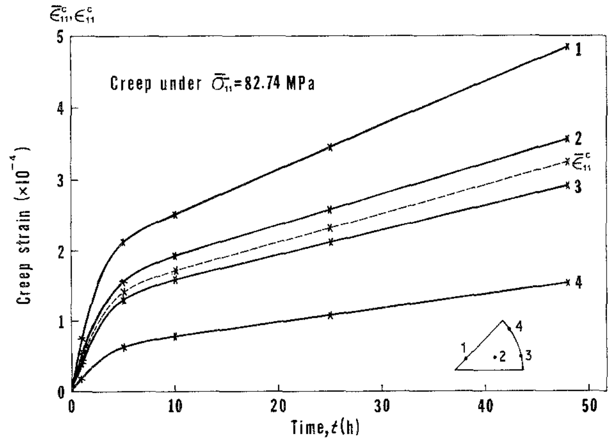

Figure 6 Creep-strain distribution among constituent grains during a tensile creep.

the test data. The same material was also tested under a combined $\bar{\sigma}_{12} = 55.5$ MPa (8.05×10^3 p.s.i.) and $\bar{\varepsilon}_{11} = 0.25\%$ (other $\bar{\sigma}_{ij} = 0$), and the measured shear creep and tensile relaxation are reproduced in Fig. 5. Also included in this figure are the theoretical calculations of combined creep and relaxation. Though the relaxation behaviour in both cases appear to be overestimated, such a deviation, in view of the overall agreement and the complicated nature of the problem, seems to lie within a tolerable range.

The problem of stress redistribution among the constituent grains, as is evident from Equations 5, 11, or 14 and 15, is directly related to the creep strain distribution. Indeed it is fundamentally interesting to see how heterogeneous creep deformation takes place in a polycrystalline aggregate. To illustrate such a heterogeneity, we depicted in Fig. 6 the tensile creep strains of four selected grains, whose orientations are shown in the inset of stereographic projections, under $\bar{\sigma}_{11} = 82.74$ MPa. Also included here for reference is the creep curve of the polycrystal. While grain orientations 1 and 2 are favourable for deformation, grain orientations 3 and 4 are not. As a consequence, though their stresses are initially equal, those of the former group, as shown in Fig. 7, continue to decrease whereas those of the latter keep increasing. Under the tensile relaxation $\bar{\varepsilon}_{11} = 0.29\%$, the corresponding creep deformation and stress distribution are depicted in Figs 8 and 9, respectively. The heterogeneous nature of creep deformation and stress relaxation among the constituent grains are again vividly displayed, the average of their behaviour giving rise to that of the aggregate.

Despite these generally satisfactory results one must bear in mind that the present study has taken the stress and strain of each grain orientation to be uniform, and that it has assumed the glide motion of dislocations, modelled as crystallographic slip, to be the sole source of creep strain. As grain deformation is usually heterogeneous and there are many other possible mechanisms of creep, these assumptions may need some qualifications. Heterogeneous deformation in a grain usually takes place near the grain boundary; it is primarily the result of compatibility accomodation from one grain to the other and of the irregular grain shape. While it is desirable to account for the variation of stress and strain within a constituent grain, it is computationally difficult. Moreover, since the fluctuation of grain shape or size, and the problem of compatibility, actually occur with equal probability to all grain orientations, it becomes reasonable to take the orientation of a grain as the determining factor leading to the stress and strain inhomogeneity from one grain to the other. The derived uniform stress and strain therefore should be interpreted as the averaged values over the heterogeneity of each grain orientation.

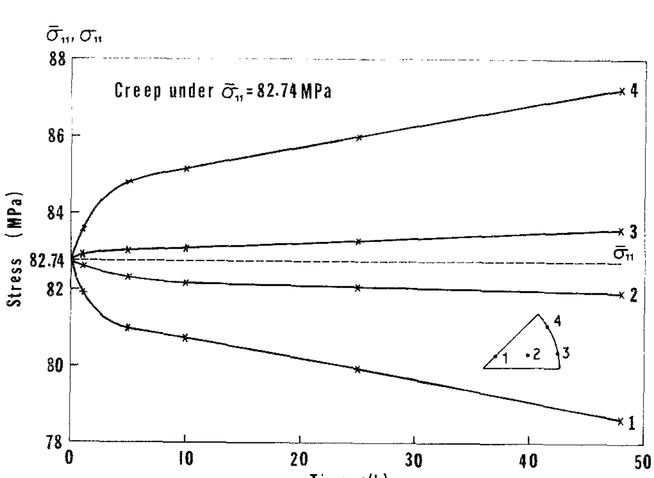

Figure 7 Stress redistribution among constituent grains during a tensile creep.

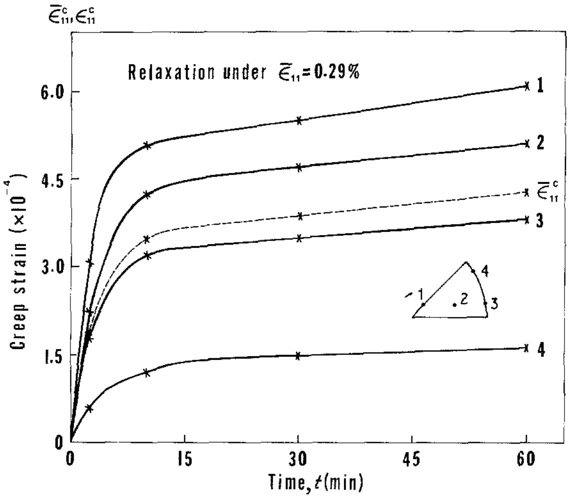

Figure 8 Creep-strain distribution among constituent grains during a tensile relaxation.

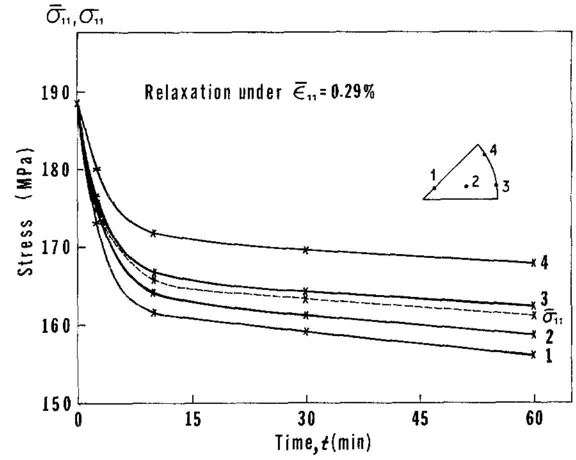

Figure 9 Stress redistribution among constituent grains during a tensile relaxation.

Although there are several mechanisms which may contribute to the creep strain of a polycrystal, their relative contributions, as indicated for instance in Frost and Ashby's deformation mechanism maps [14], depend on the considered stress and temperature state. At high stress and lower temperature the glide motion of dislocation is indeed the main source of creep strain. But as the temperature increases and especially at a low stress, Nabarro–Herring creep and Coble creep may become more dominant. Grain-boundary sliding is also easier at higher temperature. The effects of these latter mechanisms to the creep and relaxation behaviour of a polycrystal apparently have not been considered in the present study; the suggested method therefore should be used in the high stress and lower temperature range.

Acknowlegement
This work was supported by the US National Science Foundation through Grant MEA-8317887.

References
1. G. J. WENG, *J. Appl. Mech.* **48** (1981) 41.
2. *Idem, Acta Metall.* **31** (1983) 207.
3. J. D. ESHELBY, *Proc. R. Soc. London,* **A241** (1957) 376.
4. E. KRÖNER, *Acta Metall.* **9** (1961) 155.
5. B. BUDIANSKY and T. T. WU, Proceedings of the 4th US National Congress of Applied Mechanics (ASME, New York, 1962) p. 1175.
6. G. M. BROWN, *J. Mech. Phys. Solids* **18** (1970) 367.
7. G. J. WENG, *J. Appl. Mech.* **48** (1981) 779.
8. W. A. WOOD and S. McK. COUSLAND, *Proc. Inst. Mech. Eng.* **178** (1963) 3-25.
9. M. RONAY, *J. Inst. Met.* **94** (1966) 392.
10. G. I. TAYLOR, *ibid.* **62** (1938) 307.
11. G. J. WENG, *Int. J. Solids Struct.* **15** (1979) 861.
12. J. L. DING and W. N. FINDLEY, *J. Appl. Mech.* **51** (1984) 125.
13. J. S. LAI and W. N. FINDLEY, *ibid.* **49** (1982) 19.
14. H. J. FROST and M. F. ASHBY, "Deformation Mechanism Maps" (Pergamon Press, Oxford, 1982).

Received 30 December 1985
and accepted 4 September 1986